$\sin^2 x = \frac{1}{2}(1 - 2\cos 2x)$
$= \frac{1}{2}(1 - (1 - 2\sin^2 x))$
$= \frac{1}{2}(2\sin^2 x)$
$= \sin^2 x$

Math 1470 Test ~~1~~ 2 Review Name Todd & Jenna & Dayna

1) Verify that the following is an identity:

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ This is an identity - graphs do ~~not~~ match! See above.

2) Verify that the following is an identity:

$(\sin x + \cos x)^2 = 1 + 2\sin x$ Not an identity - graphs do not match

3) Write as a sum (you can verify by graphing):

$y = \sin 3m \cdot \cos m$

$= \frac{1}{2}[\sin 4m + \sin 2m]$

4) Verify that the following is an identity:

$\frac{\sin 2t + \sin 4t}{\cos 2t - \cos 4t} = \cot t$

See next page

$\frac{2\sin 3t \cos t}{2\sin 3t \sin t} = \frac{\cos t}{\sin t} = \cot t$

5) Solve the following to four decimal places:
$2\sin x = \cos 2x$

on $[0, 2\pi)$: 0.3747, 2.7669

6) Solve the triangle with sides:
$a = 4m, b = 10.2m, c = 9.05m$

$B = \cos^{-1}\left(\frac{10.2^2 - 9.05^2 - 4^2}{-2(9.05)(4)}\right)$

$B = 94.86°$

$C = 62.14°$

$A = 23°$

$\frac{4}{\sin A} = \frac{10.2}{\sin 94.86} \Rightarrow A = 23°$

$+94.86$

$180 - 117.86 = 6$

7) Solve the triangle with the following:
$\alpha = 122°, \gamma = 18°, b = 12km$

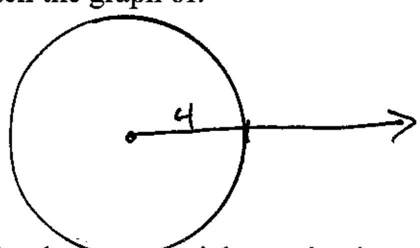
$180 - (122 + 18) = 40°$

$\beta = 40°$ $a = 15.83km$

$\frac{c}{\sin 18} = \frac{12}{\sin 40} = \frac{a}{\sin 122}$ $c = 5.77km$

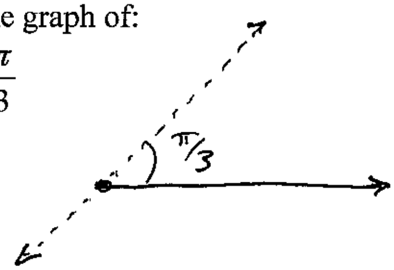
8) Sketch the graph of:

$r = 4$



10) Sketch the graph of:

$\Theta = \frac{\pi}{3}$



11) Write the exponential equation in logarithmic form: $32^{2/5} = 4$

$\log_{32} 4 = 2/5$

12) Solve the equation for x.

$e^{x^2 + 8} = e^{6x}$

$x^2 + 8 = 6x$

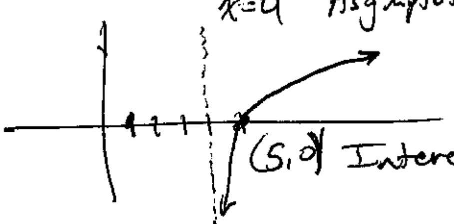
$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 4$ or $x = 2$

13) Find the domain, intercepts, and asymptotes of the logarithmic function **and** sketch its graph:
$h(x) = \log_2(x - 4)$

$x = 4$ Asymptote



$(4, \infty)$ Domain

$(-\infty, \infty)$ Range

$(5, 0)$ Intercept

14) Approximate the logarithm using the properties of logarithms, given $\log_b 2 = 0.3562$ and $\log_b 3 = 0.5646$.

$\log_b(3/4) = \log_b 3 - \log_b 4$

$= \log_b 3 - \log_b 2^2$

$= \log_b 3 - 2\log_b 2$

$= 0.5646 - 2(0.3562)$

$= -0.1478$

#4) $\dfrac{\sin 2t + \sin 4t}{\cos 2t - \cos 4t} = \cot t$

Double angle formulas $(4t = 2\cdot 2t)$

$\dfrac{\sin 2t + 2\sin 2t\cos 2t}{\cos 2t - (\cos^2 2t - \sin^2 2t)} = \cot t$

Double angle formula (again)

$\dfrac{\sin 2t(1 + 2\cos 2t)}{\cos 2t - \cos^2 2t + \sin^2 2t} = \cot t$

$\dfrac{2\sin t\cos t(1 + 2(\cos^2 t - \sin^2 t))}{\cos^2 t - \sin^2 t - (\cos^2 t - \sin^2 t)^2 + (2\sin t\cos t)^2} = \cot t$

$\dfrac{2\sin t\cos t(1 + 2\cos^2 t - 2\sin^2 t)}{\cos^2 t - \sin^2 t - (\cos^4 t - 2\cos^2 t\sin^2 t + \sin^4 t) + 4\sin^2 t\cos^2 t} = \cot t$

$\dfrac{2\sin t\cos t(1 + 2\cos^2 t - 2\sin^2 t)}{\cos^2 t - \sin^2 t - \cos^4 t - \sin^4 t + 6\sin^2 t + \cos^2 t} = \cot t$

$\dfrac{2\sin t\cos t(1 + 2\cos^2 t - 2\sin^2 t)}{1 - \sin^2 t - \sin^2 t - (1 - \sin^2 t)^2 - \sin^4 t + 6\sin^2 t + (1 - \sin^2 t)} = \cot t$

$\dfrac{2\sin t\cos t(1 + 2\cos^2 t - 2\sin^2 t)}{-8\sin^4 t + 6\sin^2 t} = \cot t$

$\dfrac{\cancel{2\sin t}(\cos t(1 + 2\cos^2 t - 2\sin^2 t))}{\cancel{2}\sin^{\cancel{2}} t(-4\sin^2 t + 3)} = \cot t$

$\dfrac{\cos t(1 + 2(1 - \sin^2 t) - 2\sin^2 t))}{\sin t(-4\sin^2 t + 3)} = \cot t$

$\dfrac{\cos t(3 - 2\sin^2 t - 2\sin^2 t)}{\sin t(3 - 4\sin^2 t)} = \cot t$

$\dfrac{\cos t}{\sin t}\dfrac{(\cancel{3 - 4\sin^2 t})}{(\cancel{3 - 4\sin^2 t})} = \cot t$

$\dfrac{\cos t}{\sin t} = \cot t$ ▨

There must be an easier way!! :)

15) Condense the expression to the logarithm of a single quantity.
3 ln(7) + 5 ln(z - 9) $\ln(7)^3 + \ln(z-9)^5 = \ln(7^3(z-9)^5)$

16) Convert the angle measure 65° from degrees to radians. Round to three decimal places.

$\frac{65}{180} = \frac{x}{\pi} \Rightarrow x = \frac{65\pi}{180} \approx 1.13$

17) Find three values for θ that make the statement true:

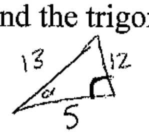
$\vartheta = (\frac{1}{2}, \frac{\sqrt{3}}{2})$ $\frac{\pi}{3}, \frac{7\pi}{3}, -\frac{5\pi}{3}$

18) Find the exact values for:

$\sin\frac{5\pi}{2} = 1$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

19) Use the given function values and the trigonometric identities to find the exact value of each indicated trigonometric function.



$\sin(\alpha) = \frac{12}{13}$

$\sin(\alpha) = \frac{12}{13}$ $\cos(\alpha) = \frac{5}{13}$

$\tan(\alpha) = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$ $\cot(\alpha) = \frac{5}{12}$

$\csc(\alpha) = \frac{13}{12}$ $\sec(\alpha) = \frac{13}{5}$

20) Solve exactly for all values of Theta on [-2π,2π] where $\sin\Theta = 1$

$\{-\frac{3\pi}{2}, \frac{\pi}{2}\}$

21) Solve exactly for all values of Theta on [-2π,2π] where $\tan\Theta = -1$

$\{-\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}\}$

22) State the period and amplitude for y=4sin3x

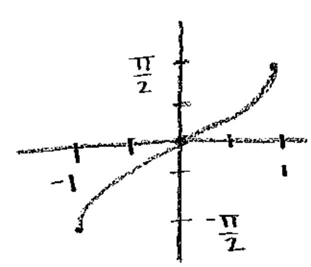
Period: $\frac{2\pi}{3}$

Amplitude: 4

23) Sketch the graph of y=arcsinx and give the domain and range.

Domain: [-1, 1]

Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$



24) Verify the identity:

$$\frac{\cos^2(\frac{\pi}{2}-x)}{\cos(x)} = \sin(x)\tan(x)$$

$\frac{\sin^2 x}{\cos x} = \frac{\sin x \sin x}{\cos x} \Rightarrow \tan x \sin x$

25) State the quadrant in which θ lies when: $\sin\theta < 0$ and $\cos\theta > 0$

$(x, -y)$

IV

Use a calculator to evaluate the trigonometric function. Round your answer to four decimal places.

$-\sqrt{2}$

sec 225° = -1.4142

26) Verify the identity:
$9\cos(t) + 9\sin(t)\tan(t) = 9\sec(t)$

$9(\cos t + \sin t \frac{\sin t}{\cos t}) = \quad 9(\frac{1}{\cos t}) = 9\sec t$

$9(\frac{\cos^2 t + \sin^2 t}{(\cos t)}) =$

27) Verify the identity (Hint: try factoring):
$9\cos^2\beta - 9\sin^2\beta = 9 - 18\sin^2\beta$

$9(\cos^2\beta - \sin^2\beta) =$

$9(1 - \sin^2 x - \sin^2\beta) =$

$9(1 - 2\sin^2\beta) = \quad 9 - 18\sin^2\beta$

28) Approximate (to three decimal places) the solutions of the equation in the interval $[0, 2\pi)$.

$4\tan^2(x) = 19$

$\tan^2 x = 19/4$

$\tan x = \pm\sqrt{19}/2$

$\arctan \sqrt{19}/2 = x$

$x = 1.141, 2.001, 4.282, 5.143$

29) Solve the equation.

$\frac{10}{5}\cos^2(x) + \frac{5}{5}\cos(x) - \frac{5}{5} = \frac{0}{5}$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\quad$ $\cos x + 1 = 0$

$\cos x = 1/2$ $\quad$ $\cos x = -1$

$\pi/3 + 2\pi n$ $\quad$ $\pi + 2\pi n$

$5\pi/3 + 2\pi n$

30) Verify the identity.

$\sec(x) - \cos(x) = \sin(x)\tan(x)$

$\frac{1}{\cos x} - \cos x = \quad \frac{1 - \cos^2 x}{\cos x} =$

$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \quad \frac{\sin^2 x}{\cos x} = \frac{\sin x \sin}{\cos x}$

$\tan x \sin x = \sin x \tan x$

31) Find the exact values of the sine, cosine, and tangent of the angle. $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$ $(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

$\sin\frac{11\pi}{12} = \sin 3\pi/4 \cos\pi/6 + \cos 3\pi/4 \sin\pi/6$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \quad \boxed{\frac{1}{4}(\sqrt{6}-\sqrt{2})}$

$\cos\frac{11\pi}{12} = \cos\frac{3\pi}{4}\cos\pi/6 - \sin 3\pi/4 \sin\pi/6$

$= -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{1}{4}(-\sqrt{6}-\sqrt{2})}$

$\tan\frac{11\pi}{12} = \frac{\tan 3\pi/4 + \tan\pi/6}{1 - \tan 3\pi/4 \tan\pi/6}$

$= \frac{1 + \sqrt{3}/3}{1 - 1(\sqrt{3}/3)} \times \frac{3}{3} = \frac{3+\sqrt{3}}{3-\sqrt{3}}$

$\frac{3+\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{12+6\sqrt{3}}{9-3} = \boxed{2+\sqrt{3}}$

32) Solve for a triangle given

a = 51 m $\quad$ A = 110.76°

b = 19.10 m $\quad$ B = 20° 30'

c = 41m $\quad$ C = 48.73°

$b^2 = 51^2 + 41^2 - 2(51)(41)\cos 20.5$

$b^2 = 364.84$

$b = 19.10$

$\cos A = \frac{19.10^2 + 41^2 - 51^2}{2(19.1)(41)}$

$\cos A = -.3545$

$A = 110.76$